because the data of Table 2 may be approximated quite accurately by straight-line relations.

With such straight-line fits of data for a given value of K, it seems possible to correlate the data to an accuracy that is well within the accuracy of the physical properties equations and the assumption of Lewis number of unity. Corrections for Lewis number $\neq 1$ which are discussed in Ref. 5 will improve the accuracy of the desired relation to the point where further refinements may not be worthwhile.

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Viscous Blunt-Body Flow with Radiation

WILHELM SCHNEIDER*
DFVLR—Institut für Theoretische Gasdynamik, Aachen, Germany

Problem and Results

THE following analysis is intended to clarify the controversial question[1,2] how radiative emission effects the flow in the shock structure of a low-density hypersonic flow around a blunt body. Bush's treatment[3] of the viscous blunt-body problem by the method of matched asymptotic expansions is extended to the flow of a radiating gas. The result is that the radiation term of the energy equation is a small, higher-order term in the entire shock structure (consisting of three different regions) if the radiation-convection parameter Γ † is of order one; in the shock layer, however, the radiation term is then of the same order of magnitude as the largest convection term. It is, therefore, correct to omit the radiation term in the shock structure provided that the limiting case of very strong radiation is excluded. Consequently Cheng's modified shock relations[4] may be used as outer boundary conditions for the viscous and radiating shock layer, and the shock layer equations can be solved independently of the solution of the shock structure.[5,6] This does not mean, however, that there is no first-order effect of radiation on the shock structure; for, an indirect influence of radiation on the shock structure arises from matching the shock structure solution with the shock layer solution.

Analysis

Nondimensional variables are used, the notation being the same as in Bush's paper[3]; x and y are boundary-layer coordinates, u and v are the velocity components in x and y direction, r is the distance from the axis of symmetry, $\varkappa$ the longitudinal curvature of the axisymmetric body, Φ the inclination angle of the body surface, and $h = 1 + \varkappa y$. All lengths are referred to the body nose radius a, all flow quantities to their freestream values. A perfect gas is assumed, having constant specific heats, constant Prandtl number Pr (of order 1) and viscosity μ varying as T^{ω}. The equations of continuity and momentum are not changed by radiative energy transfer, but the equation of energy, as given by Bush,[3] has to be completed by adding a radiation term. Because of the low density, absorption within the shock layer and shock structure is negligible.[6] The energy emitted per unit time and unit mass is denoted by Q^*, and a nondimensional emission rate $Q = Q^*/Q_{\max}^*$ is introduced, with $Q_{\max}^*$ being the maximum value of Q^* in the flowfield. Now the energy equation can be written as

$$\rho\left(\frac{u}{h}\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - \frac{2\epsilon}{1+\epsilon}\left(\frac{u}{h}\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) = \frac{1}{PrRe}\left[\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right) + \left(\frac{\varkappa}{h} + \frac{\cos\Phi}{r}\right)\mu\frac{\partial T}{\partial y} + \frac{1}{h}\frac{\partial}{\partial x}\left(\frac{\mu}{h}\frac{\partial T}{\partial x}\right) + \frac{\sin\Phi}{r}\frac{\mu}{h}\frac{\partial T}{\partial x}\right] + \frac{2\epsilon M^2\mu}{(1-\epsilon)Re}\times \left[2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{1}{h}\frac{\partial u}{\partial x} + \frac{\varkappa v}{h}\right)^2 + 2\left(\frac{u\sin\Phi + v\cos\Phi}{r}\right)^2 + \left(\frac{\partial u}{\partial y} - \frac{\varkappa u}{h} + \frac{1}{h}\frac{\partial v}{\partial x}\right)^2 - \frac{2}{3}\left(\frac{1}{h}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\varkappa v}{h} + \frac{u\sin\Phi + v\cos\Phi}{r}\right)^2\right] - \frac{2\epsilon M^2}{1-\epsilon}\Gamma\rho Q \quad (1)$$

where M is the freestream Mach number, Re the freestream Reynolds number, $Re = \rho_\infty U_\infty a/\mu_\infty$, Γ a radiation-convection parameter, $\Gamma = aQ_{\max}^*/U_\infty^3$,‡ and $\epsilon = (\gamma-1)/(\gamma+1)$ with γ as ratio of the specific heats.

For the case without radiation, $\Gamma = 0$, asymptotic expansions were constructed by Bush[3] for $M \to \infty$, $Re \to \infty$, and $\epsilon \to 0$ in such a way that $\delta = (1-\epsilon)/2\epsilon M^2 \to 0$. In order to extend the expansions to the flow of a radiating gas, we notice that the orders of magnitude of the flow variables will not be changed by the effects of radiation, provided that the case of very strong radiation ($\Gamma \to \infty$) is excluded from the considerations. Hence we can take over Bush's matched asymptotic expansions into the analysis of the radiating field, and the extension is made by simply expanding the emission term of the energy equation (1) in the same manner as the flow variables. In the course of these developments, it will be assumed that Q^*, i.e., the energy emission per unit mass, decreases with decreasing temperature at least as $T^{1-\omega}$, and that Q^* also decreases with decreasing density or that is almost independent of the density. These assumptions are usually valid for nonequilibrium radiation[6] as well as in the case of local thermodynamic equilibrium.[8] The results of the expansions for the various flow regions are given below, and the magnitude of the radiation term, denoted by q_{rad}, is compared with the magnitude of a typical leading term, which may be either a convection term q_{conv}, or a conduction term q_{cond}.

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* Research Scientist.

† Γ is defined subsequent to Eq. (1).

‡ If the gas were in local thermodynamic equilibrium, Γ would be equivalent to the parameter $\Gamma_s\alpha_s\epsilon a$ used by Liu and Sogame.[7]

Outer region of the shock structure

The coordinates and the expansions are

$$x = \xi_0; \quad y = \epsilon Y_0(\xi_0) + \eta_0/Re$$

$$u = \cos\Phi + \delta^{3/4Pr}u_0 + \dots; \quad v = -\sin\Phi + \delta^{3/4Pr}v_0 + \dots \quad (2)$$
$$\rho = 1 + \delta^{3/4Pr}\rho_0 + \dots; \quad p = p_0 + \dots; \quad T = T_0 + \dots$$

where ξ_0, η_0, u_0, ... are of order one. Since the maximum temperature in the flowfield is $O(T_0/\delta)$, cf. Eqs. (6, 10, and 13), the nondimensional emission rate Q is $O(\delta^{1-\omega})$ or smaller. Hence the radiation term of Eq. (1) can be written as

$$q_{\text{rad}} = -\delta^{-1}\Gamma\rho Q \leqq O(\Gamma\delta^{-\omega}) \quad (3)$$

For comparison, typical convection and conduction terms, respectively, are

$$q_{\text{conv}} = \rho v(\partial T/\partial y) = -Re\,\sin\Phi(\partial T_0/\partial\eta_0) + \dots \quad (4)$$

$$q_{\text{cond}} = (\mu/PrRe)(\partial^2T/\partial y^2) = RePr^{-1}T_0^{\omega}(\partial^2T_0/\partial\eta_0^2) + \dots$$

It follows that

$$(q_{\text{rad}}/q_{\text{conv}})_0 = O[(q_{\text{rad}}/q_{\text{cond}})_0] \leqq O(\Gamma/\delta^{\omega}Re) \to 0 \quad (5)$$

since $(\delta^{\omega}Re)^{-1} \leqq O(\epsilon) \to 0$ as has been shown by Bush.[3]

Middle region of the shock structure

$$x = \xi_m; \quad y = \epsilon Y_m(\xi) + \eta_m/\delta^{\omega}Re$$

$$u = u_m + \dots; \quad v = v_m + \dots; \quad \rho = \rho_m + \dots \quad (6)$$

$$p = p_m/\delta + \dots; \quad T = T_m/\delta + \dots; \quad Q = Q_m + \dots$$

Expanding the radiation term of Eq. (1) gives

$$q_{\text{rad}} = -\Gamma\delta^{-1}\rho Q = -\Gamma\delta^{-1}\rho_m Q_m + \dots \quad (7)$$

Typical convection and conduction terms, respectively, are

$$q_{\text{conv}} = \rho v \frac{\partial T}{\partial y} = \delta^{\omega-1}Re\rho_m v_m \frac{\partial T_m}{\partial \eta_m} + \dots$$
$$q_{\text{cond}} = \frac{\mu}{PrRe}\frac{\partial^2 T}{\partial y^2} = \delta^{\omega-1}RePr^{-1}T_m^{\omega}\frac{\partial^2 T_m}{\partial \eta_m^2} + \dots \quad (8)$$

Hence

$$(q_{\text{rad}}/q_{\text{conv}})_m = O([q_{\text{rad}}/q_{\text{cond}}]_m) = O(\Gamma/\delta^{\omega}Re) \to 0 \quad (9)$$

Inner region of the shock structure

$$x = \xi_i; \quad y = \epsilon Y_i(\xi_i) + (\epsilon/\delta^{\omega}Re)\eta_i$$

$$u = W(\xi_i) + \epsilon u_i + \dots; \quad v = \epsilon v_i + \dots;$$
$$\rho = \rho_i/\epsilon + \dots \quad (10)$$
$$p = p_i/\epsilon\delta + \dots; \quad T = \delta^{-1}[\theta(\xi_i) + \epsilon T_i + \dots];$$
$$Q = Q_i + \dots$$

The radiation term of Eq. (1) becomes

$$q_{\text{rad}} = -\Gamma\epsilon^{-1}\delta^{-1}\rho_i Q_i \quad (11)$$

and, since $q_{\text{conv}}/q_{\text{cond}} = O(\epsilon) \to 0$ in this layer, q_{rad} has to be compared with the leading conduction terms, yielding

$$(q_{\text{rad}}/q_{\text{cond}})_i = O(\Gamma/\delta^{\omega}Re) \to 0 \quad (12)$$

Shock layer

$$x = \xi; \quad y = \epsilon\eta_L$$

$$u = u_L + \dots; \quad v = \epsilon v_L + \dots; \quad \rho = \rho_L/\epsilon + \dots \quad (13)$$

$$p = p_L/\epsilon\delta + \dots; \quad T = T_L/\delta + \dots; \quad Q = Q_L + \dots$$

The radiation term can be written as

$$q_{\text{rad}} = -\Gamma\epsilon^{-1}\delta^{-1}\rho_L Q_L + \dots \quad (14)$$

Here the conduction terms are smaller or of the same order of magnitude as the leading convection terms. Thus, we compare the radiation term with a typical convection term to obtain

$$(q_{\text{rad}}/q_{\text{conv}})_L = O(\Gamma) = O(1) \quad (15)$$

The Eqs. (5, 9, 12, and 15) form the result that has already been discussed in the first section.

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Flutter of a Buckled Plate Exposed to a Static Pressure Differential

C. S. VENTRES*
Princeton University, Princeton, N.J.

HESS[1] has recently published an experimentally determined stability boundary for a panel exposed simultaneously to a static pressure differential and to a streamwise applied in-plane load. Prior efforts in this direction were either limited to panels free of applied in-plane loads,[2] or were carried out in a manner that did not provide precise control of the in-plane load, because of limitations inherent in the type of wind tunnel used.[3,4] In this Note, Hess' experimental results will be compared with a theoretical stability boundary calculated by the present author, using previously developed theoretical techniques.[5–7] The work represents an extension of previous work on flutter boundaries of pressure loaded plates free of externally applied in-plane loading.[5–7]

The panel tested by Hess had a length-width ratio of 2.88. The tunnel Mach number was 1.96. The panel flutter boundaries and flutter frequencies were determined at various values of static pressure differential and in-plane load. At each static pressure level, the in-plane load was varied until the minimum flutter dynamic pressure was realized. This minimum flutter dynamic pressure and the associated flutter fre-

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* Member of the Research Staff.